Functions and Inverse Functions

1.0 Objectives

This chapter covers the following topics related to functions. After successful completion of this section, you will be able to:

- Define a function and determine the domain and range of functions.
- Define what a one-to-one function is and calculate the inverses of such functions.
- Give examples of common functions and their inverses.
- Manipulate expressions and equations involving logarithms and exponentials.
- Define and graph logarithmic and exponential functions.
- Manipulate expressions and equations, and graph functions such as trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions.

1.1 Introduction to Functions: Finding the domain and range of a function

The following video provides an introduction to finding the domain and range of a given function for various graphs.

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/KirGQOwjBVI`

1.2 Domain and range of a function

1) The following video demonstrates how to find the domain for the functions listed below

a) $f(x) = \dfrac{x+3}{x-7}$

b) $f(x) = \dfrac{2x+1}{x^2-16}$

c) $f(x) = \dfrac{x^2}{x^2+9}$

e) $f(x) = \sqrt{x-9}$

f) $f(x) = \sqrt{x^2+5x-21}$

g) $f(x) = \sqrt[3]{-x+5}$

i) $f(x) = \dfrac{\sqrt{x+5}}{x^2-5}$

j) $f(x) = \dfrac{3x-4}{\sqrt{x+9}}$

k) $f(x) = \dfrac{\sqrt{x+1}}{\sqrt{x^2-36}}$

2) The following video demonstrates how to find the domain and range for the following functions.

(i) Linear functions

a) $f(x) = x + 3$

(ii) Quadratic functions

a) $f(x) = x^2$ b) $f(x) = x^2 - 3$ c) $f(x) = 4 - x^2$
d) $f(x) = x^2 - 4x + 5$

(iii) Cubic functions

a) $f(x) = x^3$ b) $f(x) = x^3 + 5x^2 - 8$ c) $f(x) = 4 - x^2$

(iv) Absolute (modulus) functions

a) $f(x) = |x|$ b) $f(x) = |x| + 3$ c) $f(x) = |x - 2| - 3$

d) $f(x) = 2 - |x - 3|$

(v) Radical functions (Functions with roots)

a) $f(x) = \sqrt{x}$ b) $f(x) = -\sqrt{x}$ c) $f(x) = \sqrt{-x}$

d) $f(x) = -\sqrt{-x}$ e) $f(x) = -\sqrt{x-3} + 4$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/Si2vmzUWfJE`

1.3 Functions and relations

The following video demonstrates the distinction between function and relationship. It also explains the types of functions, such as one-to-one and many-to-one.

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/6u9Qh1Vi_Qw`

1.4 Functions and their inverses

The following video demonstrates the relationship between functions and their inverses, along with the following worked examples.

(i) Linear functions a) $f(x) = 7x + 2$ b)$f(x) = 8x - 5$ c) $f(x) = \frac{x}{5} + 1$

ii) Quadratic functions

a) $f(x) = x^2 + 4$

iii) Radical functions

a) $f(x) = \sqrt{2x^3 + 58}$

iv) Inverse composite functions

a) $f(x) = x + 5$ and $g(x) = x^2 - 2$, workout $(gf)^{-1}(x)$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/1aBkyMVvlsU`

1.5 Exponential and Logarithmic Functions

The videos included in the links below demonstrate expressions and equations involving logarithms and exponents. They also define and graph logarithmic and exponential functions, as well as find their domain and range.

i) The link below demonstrates how to find the inverse of the following logarithmic functions.

a) $f(x) = \log_2(x - 3) - 5$ b) $f(x) = 2\log_4(x - 5) + 12$

c) $f(x) = \ln(x - 3) + 4$ d) $f(x) = 5 - \ln(6 - \frac{1}{3}x)$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/hNsvGz7JPJQ`

ii) The link below demonstrates how to find the inverse of the following exponential functions

a) $f(x) = 3^x + 5$ b) $f(x) = 2(4^x) - 10$

c) $f(x) = 4e^x - 20$ d) $f(x) = 8 - 2e^{5x}$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/ihkYZOwPetM`

1.6 Trigonometric and hyperbolic functions, their graphs, inverses, equations, and identities

i) The following video demonstrates the relationship between the standard trigonometry functions and their inverse.

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/YXWKpgmLgHk`

ii) The following videos demonstrate sketching the inverse trigonometric functions listed below.

a) $y = sin^{-1}(x)$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/YNstP0ESndU`

b) $y = cos^{-1}(x)$

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/YNstP0ESndU`

iii) The following videos demonstrate various aspects of hyperbolic functions, including graphing and solving identities.

a) Introduction to hyperbolic functions

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/PJRSu0Vf0r0`

b) Graphs of hyperbolic functions

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/w_UEjfADQQc`

c) Graphs of inverse hyperbolic functions

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/TaVdqXsSksA`

d) Hyperbolic identities

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/m9nwdn55Z2w`

e) Solving hyperbolic equations using identities

Video Visit the URL below to view a video:

`https://www.youtube.com/embed/gIZwP5r08Rk`